I. INTRODUCTION

Chapter 1: A Quick First Look – The Context

Look at the Earth from the perspective of an alien visitor to our solar system. The outer planets are gassy giants, nearly Suns themselves. The hard-surfaced moons of these planets are intensely cratered. Some of the moons have deformable (Ganymede) or tectonically active surfaces (Io) and are able to erase the evidence of past impacts, but others, such as the moon, record a history of bombardment over more than 4000 million years (4Ga).

The inner hard-surfaced planets are more like the moons of the outer gassy giants than the gassy giants themselves. The differences between the three planets within habitable distance from the Sun are striking. Mars is cold (polar caps of dry ice) and windy. Venus looks attractive but is much too hot. Its runaway greenhouse effect has raised its surface temperature over 400K to 737K (864F), well above the melting point of lead (500K). Most of the Earth has just the right temperatures to support life; it is a beautiful planet largely covered by oceans. Although some evidence of meteorite impact can be found, such as Meteor Crater, in Arizona and Manacougan Crater, in Quebec, the Earth shows remarkably few obvious signs of meteorite bombardment. This is because tectonic processes constantly produce new surface and erosion erases or mutes what evidence survives. The odd and very systematic variations of surface elevation in the oceans and continents are immediately noted. Also night-lights show the planet is densely populated by an intelligent species that is utilizing the resources of the planet in a somewhat wasteful fashion (slash and burn fires in the Amazon and Africa, the flaring of gas in Nigeria and the Arabian Peninsula).
Chapter 2: A Closer Look - Some Energetics

The masses of the planets and observations of meteorites and comets reveals a great deal about how the Earth was born. The mass of the planets is indicated by the revolution periods and mean orbital radii of their moons, both of which can be observed from Earth (Kepler's Laws, see Page 4). The densities of the inner planets are \( \sim 5 \text{ Mg/m}^3 \) (or, equivalently, \( \text{tonne/m}^3 \) or \( \text{g/cc} \)). The densities of the outer planets are \( \sim 1.3 \text{ Mg/m}^3 \). The outer surface of the Earth has a density of \( \sim 2.7 \) to \( 3 \text{ Mg/m}^3 \), which even if compressed by the pressures in the Earth’s interior, could not account for the average \( 5.52 \text{ Mg/m}^3 \) density of the Earth. This indicates that the Earth is a chemically differentiated planet.

Comets are largely composed of frozen gas, including a lot of ice of density \( \sim 1 \text{ Mg/m}^3 \). Stony meteorites have a density of \( 3.5 \text{ Mg/m}^3 \). Iron meteorites have densities of \( \sim 7.5 \text{ Mg/m}^3 \). Thus, based on density considerations alone, the inner planets seem to be composed of a mix of stony and iron meteorite material, and the outer planets largely of cometary gases.

Closer examination of meteorites provides more information. There are three kinds of stony meteorites: chondrites, carbonaceous chondrites, and achondrites. Chondrites are formed from condrules, \( \sim 1 \text{ mm spherules that were once molten. The chondrules agglomerated with other material in space. Some of the chondrites contain carbonaceous material mixed in with the chondrules. Some carbonaceous chondrites contain material that could never have been heated above \( \sim 373K \) (100°C). From the absorption band intensities in the Sun’s radiation spectrum, we know that the ratios of elements in carbonaceous chondrites are practically identical (within measurement error) to the ratios of elements in the Sun. The recent Galileo probe’s measurements of Jupiter’s outer atmosphere also found a gas composition similar to that of the sun. For these reasons it is thought that the Sun, planets, and carbonaceous chondrites accreted from the same material.

Initially there was a lot of cometary gas associated with the carbonaceous chondrite material that accreted to form the planets and Sun. As the Sun heated up, the solar wind blew the gas from the accreting inner planets. The inner planets have a residual chondrite composition and the outer planets contain chondrite and gas material in proportions more similar to that in the Sun.

The mix of elements in the planets and Sun tells us that the elements were synthesized by fusion reactions (to produce elements up to iron) and then by the intense neutron bombardment that occurs in supernova explosions (to synthesize elements heavier than iron). It is likely that agglomeration of gas and debris to form our solar system was triggered by a nearby supernova explosion. Short-lived radioactive elements produced in the supernova explosion like \( ^{26}\text{Al} \) helped the Earth heat up rapidly. Once heated sufficiently, the iron core of the Earth segregated, an event that could have released enough gravitational energy to heat the Earth to over 6000K – just by itself.

The Earth is believed to have been born of at least three gigantic explosions. (1) The original Big Bang that produced Hydrogen (H) and Helium (He) which collected in stars to synthesize elements up to iron, (2) supernova explosions whose intense neutron flux synthesized elements heavier than iron, (3) a local supernova that triggered the formation of our solar system. The Earth was formed late in the history of the universe. If it had not been late in formation, there would not have been time for elements heavier
than iron to be synthesized in supernova explosions and the debris incorporated into our solar system.

The early Earth was intensely bombarded by meteorites. Analysis of the overlapping impact craters on the moon coupled with the known age’s of parts of the moon’s surface from radioactive age-dating of samples brought back by the Apollo and Luna missions indicates that the intensity of this bombardment dropped rapidly with time as the solar system aged.

In the Problem Set 2a you will calculate the relative magnitude of some important Earth events. You will find that the energy released by segregation of the central iron core is equivalent to the energy in $8 \times 10^6$ years of Sunlight falling on the Earth. Collision of the Earth with a 20 km diameter meteorite is equivalent to about 2.5 months of Sunlight, and a very large earthquake is equivalent to about 2 minutes of Sunlight.

Because the input of solar energy at a planet’s surface is so much larger than the internal heat released from the planet’s interior, the balance between incoming solar energy and the reflected and radiated thermal energy determines its surface temperature. If Venus, Earth and Mars had the black body (a theoretically ideal absorber and radiator of energy at all electromagnetic wavelengths, where all impinging radiation is emitted back at all wavelengths) temperatures required to radiate the absorbed incident electromagnetic energy they receive from the Sun, they would have temperatures of 328K, 278K, and 228K (55, 5 and -50°C) respectively. If account is taken of the fraction of sunlight reflected from each planet (their albedo), the black body temperatures of Venus, Earth and Mars would be 244K, 253K, and 213K (-29, -20 and -60°C) respectively. The runaway greenhouse heating of Venus adds ~400K to its surface temperature and accounts for the ~730K (460°C) temperature of that planet. A 35°C greenhouse heating of the Earth gives it a habitable 15°C average temperature. For Mars, the greenhouse heating of 15K is insufficient to make it very comfortable (average T of 230K (-45°C)). Clearly we are dependent on greenhouse warming for our comfortable Earthly habitat. In fact, life may have something to do with regulating the CO$_2$ - greenhouse effect to keep Earth temperatures in a habitable range. The incineration of our neighbor Venus indicates we are right to be concerned, but we must be careful not to eliminate the greenhouse effect entirely, or we will freeze.

Additional Reading

One of the most revealing observations that can be made of our solar system is the large difference in density between the inner terrestrial planets, and the outer gassy planets. We can measure a planet’s density if its has a satellite orbiting it by using Kepler's Laws, since these relations reflect the balance between gravitational and centrifugal forces that determines the orbit of a planet or moon.

Kepler's Laws are:

1. All planets move in elliptical orbits with the Sun at one focus. The other focus is located symmetrically at the opposite end of the elliptical orbit (Figure 2.1).
2. A line joining any planet to the Sun sweeps out equal areas in equal times. As seen in Figure 2.1, if \( \Delta t_1 = \Delta t_2 \), then Area 1 = Area 2.
3. The square of the planets orbital period is proportional to the cube of the planet's mean distance from the Sun \( (T^2 = kr^3) \), where \( k = \) constant of proportionality).

![Figure 2.1 Kepler’s Laws.](image)

Kepler's Laws can be derived by balancing gravitational and centrifugal forces. If \( M \) is the mass of the Sun, \( m \) is the mass of a planet in orbit around the Sun, \( r \) is the mean orbital radius of that planet, \( G \) is the gravitational constant, and \( \omega \) is the angular velocity of the planet \( (\omega = 2\pi/T, \) where \( T \) is the period of the orbit), then:

\[
\text{Gravitational Force} = G\frac{Mm}{r^2}, \quad \text{and}
\]

\[
\text{Centrifugal Force} = m\omega^2 r = \frac{4\pi^2mr}{T^2}.
\]

Setting the two equal and canceling out the common term \( m \) gives:

Kepler's Law: \( T^2 = \frac{4\pi^2r^3}{GM} \).

Notice that the orbital period does not depend on the mass of the planet (the object in orbit), but only on the mass of the Sun (the object it orbits about). The equation gives Kepler's Third Law directly.
As an example, we can use the mathematical expression of Kepler's Third Law to determine the mass of the Sun, given the distance of the Earth from the Sun and the period of the Earth’s orbit. For example, taking $r_{e-s} = 150$ million kilometers, $G = 6.67 \times 10^{-11}$ m$^3$/kg-s$^2$ and $T_e= 1$ year ($3.15 \times 10^7$ sec), the mass of the Sun must be:

$$M_{\text{Sun}} = \frac{4\pi^2 r_{e-s}^3}{GT^2} = \frac{4\pi^2 (150 \times 10^6 \times 10^3)^3}{(6.67 \times 10^{-11})(3.15 \times 10^7)^2} = 2.01 \times 10^{30} \text{ kg.}$$

Note this actually depends on a hard thing to measure on Earth — the distance between the Earth and Sun. Kepler’s Law relates ratios of radii to ratios of periods or revolution, and 19th century astronomy used the transit of Venus across the sun to determine the Earth-Sun distance, from which all other distances could be determined.

**Problem Set 2a: Kepler’s Laws**

**Problem 1:** Calculate the mass of the Earth from the period of revolution of the moon around the Earth. The average orbital distance between the Earth and moon is $3.8 \times 10^5$ km. If the radius of the Earth is $6.37 \times 10^3$ km, what is the average density of the Earth?

**Problem 2:** Calculate the density of Jupiter given that Jupiter's equatorial radius is 71,492 km and that Ganymede, one of Jupiter's moons, orbits Jupiter at an average distance of 1.07 million km every 7.15 days.

**Problem 3:** Why is Jupiter so much less dense than the Earth?
2.2 Energy Fluxes and Events on the Early Earth

This section reviews the concepts of force, work, energy, units of measurements and also illustrates the use of dimensional analysis. Our purpose is to lay a foundation for energy calculations such as:

1. Heating all the dust and agglomerated rock fragments that collected to form the Earth - to the melting point.
2. Relative energy flux in solar radiation and meteorite bombardment.

**Force**

Force can be defined by Newton's Law:

\[ F = ma, \]

Where \( F \) is the force in Newtons (in the MKS or meter-kilogram-second system, also known as SI) or Dynes (CGS or centimeter-gram-second system), \( m \) is the mass in kilograms (MKS) or grams (CGS), and \( a \) is the acceleration in m/sec\(^2\) or cm/sec\(^2\). A Newton clearly has the the dimensions of kg-m/sec\(^2\).

*Why? Show that a Newton is \(10^5\) dynes.*

**Work (Energy)**

By definition, work or energy, \( E \), is the product of force and distance. If the force is not constant, \( E \) may be defined:

\[ E = \int_{r_1}^{r_2} F \cdot dr \]

Where \( E \) is energy in Newton-m, also called Joules, \( F \) is force in Newtons, and \( dr \) is incremental (radial) distance in m. If \( r_1 \) is taken to be \( \infty \) and \( r_2 \) the surface of a planet, \( E \) is the gravitational potential energy of a distant stationary body with respect to the surface of that planet. If that body falls to the surface of the planet, this gravitational potential energy is converted to kinetic energy. When the body hits the surface the kinetic energy is transferred to smaller objects (explosion) and ultimately converted to heat, some or much of which is radiated back into space. By definition energy is given the units of Joules (1 Joule = 1 N-m in SI or MKS).

*Show that energy has the units \( \text{kg-m}^2/\text{sec}^2 \).*
Energy Equivalents

To develop an intuitive understanding it helps to express energy in different units. The relationships between some convenient units are:

\[ 1 \text{ joule} = 1 \text{ kg-m}^2/\text{sec}^2 = 10^7 \text{ ergs} = 10^7 \text{ g-cm}^2/\text{sec}^2 \]

1 calorie = 4.186 joules  
Historical note – a calorie was defined as the amount of heat (then thought of as ‘caloric’) that would raise the temperature of 1g of water by 1°C, before it was realized that heat and work are both forms of energy. Since chemical manipulations still frequently use calories, while physical ones are in SI joules, this is one conversion that sometimes can’t be avoided. However, it is convenient that roughly a joule of energy is needed to heat a gram of silicate mantle by one degree Kelvin. (When calories were being defined, it was also not yet known how unusually high water’s heat capacity was relative to most other solids and liquids...)

1 ton TNT = 4 x 10^9 joules = 4 GJ

Energy produced by a large 100MW powerplant in one year = 3.15x10^{15} J

Largest historical earthquake ~ 10^{19} J (Chile, 1960 – Magnitude 9.5, with ~800km of surface rupture)

Solar flux at distance of Earth from Sun = 1.37 kW/m^2 (1370 J/m^2-s)

Planetary data you will find useful include:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Radius (10^6 m)</th>
<th>Mass (10^{24} kg)</th>
<th>Distance from Sun (10^{11} m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>2.44</td>
<td>0.33</td>
<td>0.58</td>
</tr>
<tr>
<td>Venus</td>
<td>6.05</td>
<td>4.9</td>
<td>1.08</td>
</tr>
<tr>
<td>Earth</td>
<td>6.38</td>
<td>6.0</td>
<td>1.50</td>
</tr>
<tr>
<td>Mars</td>
<td>3.4</td>
<td>0.64</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Newton’s Law of Gravitation

Newton’s law of gravitation is:

\[ \vec{F}_{\text{gravity}} = \hat{r} \frac{G M_1 m_2}{r^2}, \]

Where \( G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2 \) is the gravitational constant. \( M_1 \) and \( m_2 \) are the masses of the two objects attracting one another, \( r \) is the distance between the centers of
masses of the two objects, and $\hat{r}$ is a unit vector pointing from the test mass $m_2$ toward $M_1$ (Figure 2.2).

*Show that $G$ has units of $m^3/kg\cdot\sec^2$.*

For convenience, gravitational acceleration, $g$, is defined:

$$\vec{g} = \hat{r} \frac{GM_1}{r^2},$$

so that:

$$\vec{F}_{\text{gravitational}} = m_2 \vec{g}$$

This expression is a close analog to Newton's definition of force defined above. For the Earth, $g$ is close to 9.8 m/sec$^2$ from the surface to 4000 km depth.

*Calculate $\vec{g}$ for the surfaces of the Earth, Mars and Venus.*

*How can $\vec{g}$ be constant to half the radius of the Earth?* (Answer the question here in words and mathematically in problem 3 below.)
Problem Set 2b: Energy Fluxes

**Problem 1:** Answer all the questions in italics in Section 2.2 above.

**Problem 2:** Calculate the solar flux to the Earth (e.g., how much solar energy impinges on a disk of radius equal to the Earth’s at the distance of the Earth from the Sun?)

**Problem 3:** If the gravitational attraction of mass at radii (from the center of mass) greater than a point buried within a planet cancel (they do), the gravitational attraction at a point in the interior of a planet is due only to that material in the spherical shells of material below that point. In other words, if I measure gravity at a point 4000 km below the Earth’s surface (2380 km from the center of the Earth), the gravitational attraction is as if I were on a planet 2380 km in radius. Use this insight to calculate the density of the Earth below 4000 km depth, if the gravitational acceleration, $g$, equals 9.8 m/s$^2$ at that depth.

**Problem 4:** The mass of the Earth’s iron-nickel core is about 1/3 that of the entire Earth. If this metallic material was initially mixed uniformly in the newly accreted Earth and then segregated to the center of the Earth, how much gravitational energy would be released? Make a rough estimate assuming (on average) the iron-nickel mass moved through a constant gravitational field of 9.8 m/s$^2$ from 2/3 $r_e$ to 1/3 $r_e$. What would this energy release mean for the heat budget of the early Earth? If all of the energy went to heat, how much would the Earth be heated? Assume the heating was uniform and that the heat capacity of the Earth is 1 J/g-K (i.e., assume that it takes 1 Joule to warm 1 gram of Earth material by 1°C). How many millions of years of solar flux would be equivalent to the energy released in this "iron catastrophe"?

**Problem 5:** Walter and Louis Alvarez have suggested that periodic collisions of the Earth and 20 km diameter meteorites are responsible for major extinctions such as that of the dinosaurs at the Cretaceous-Tertiary boundary 65 million years ago. A 20 km diameter meteorite would create a 200 km diameter crater. Estimate the amount of energy released in such an impact by calculating the work gravity does on the meteorite as it falls from a very large distance to the Earth’s surface. Assume a meteorite density of 3000 kg/m$^3$. How many days of solar flux on Earth would equal this energy? Note that this gravitational energy would have been transformed into kinetic energy \( K.E. = \frac{1}{2} m_{\text{meteor}} v_{\text{meteor}}^2 \) of the moving meteor just before it hit. How fast would the meteorite be moving when it hit Earth’s surface? Can you think of any ways it could be moving faster (or slower?) than this?

**Problem 6:** How many minutes of Sunlight on the area 50,000 m$^2$ (the area of a large powerplant complex) would equal the energy produced by a 100MW powerplant in a year? Compare the energy (in terms of millions of years, days, or minutes of Sunlight) of the segregation of Earth’s core and the impact on Earth of a 20 km meteorite.
Problem 7: Making planets by colliding ice-cubes. The Earth is presumed to have grown by incessant collisions of smaller objects. Imagine a small world of Earth-like density, upon which small comets (ice-cubes) rain down, pulled in by the proto-Earth’s gravitational attraction. At what earth-radius would the kinetic energy release from such a collision generate more heat that the latent heat it would take to melt the impacting ice-cube? At what radius would the impact release enough energy vaporize the impacting ice-cube? (Assume each comet is composed of ice at 273K, only considering the enthalpy of melting of ice, which is 333kJ/kg. The enthalpy of vaporization of water at 373K is 2222kJ/kg. Also assume that all of the impact energy goes into heating the impacting ice-cube, not the proto-Earth)

Problem 8: Energetics of a moon-forming impact. The current conventional wisdom for the formation of the moon is that roughly ~30Ma after accretion of most of the Earth (and segregation of most of the core), a large Mars-sized planet collided with the proto-Earth, the impact leading to the formation of the moon from the ring of ejecta induced by this impact that collected at a distance greater than 3 Earth radii. Assume a Mars-mass object collides with an Earth-mass object at the speed predicted by their maximum gravitational potential energy. To what temperature would this energy heat the Earth, if after the impact, the heat were uniformly distributed throughout the proto-Earth, assuming the heat capacity of the Earth is ~1 kJ/kg.) If all of the gravitational energy release went into the impacting object, do you think it would melt or be vaporized? (The enthalpy of melting of an Earth-like silicate is ~660kJ/kg, while the enthalpy of vaporization is ~13 MJ/kg.)

Problem 9: Discuss in a paragraph or two (not more) the role that "back of the envelope" calculations such as you have carried out above play in comprehending the past and future history of the Earth? How do such calculations compare to more sophisticated and accurate computer simulations? Why are “back of the envelope” calculations still useful in the computer age?
2.3 The Black Body Temperature of Planets

After the marked density contrast between the inner and outer planets, the most surprising observation on our solar system is probably the difference in temperature between the Earth and its nearest neighbors. Venus, the next closest planet to the Sun has surface temperatures high enough to melt lead. Mars, the next farther away, has surface temperatures sufficient to make dry ice (CO\(_2\)) freeze. Are the differences simply due to the distance from the Sun? Calculating the black body temperatures of the planets allows this possibility to be evaluated and is the first step toward understanding planetary temperature.

A perfect black body radiates according to a simple law discovered by Plank:

\[ R_C = \sigma T^4 \]

Where \( R_C \) is the radiance in J/m\(^2\)-s (or, equivalently, W/m\(^2\)), \( \sigma \) is the Stephan-Boltzmann constant with a value of \( 5.65 \times 10^{-8} \) J/m\(^2\)-sec-K\(^4\), and \( T \) is temperature in degrees Kelvin (= °C + 273.15).

**Example 1:** Given that the flux of solar heat to the Earth at the mean distance of the Earth from the Sun is \( 1.37 \) kJ/m\(^2\)-s, and that the radius of the Earth is \( 6.37 \times 10^6 \) m, use Plank’s Law to calculate the black body temperature of the Earth.

Note that solar energy is intercepted by the disk the Earth presents to the Sun (area = \( \pi r_e^2 \)) whereas, because the Earth rotates, the Earth radiates energy from its full spherical surface (\( 4\pi r_e^2 \)). Thus if the Earth’s albedo (the fraction of incident energy reflected to space) is \( A \) we can write the following general equation for the black body temperature of the Earth or any other planet:

\[
T_{\text{Black Body}}[\text{°K}] = \left[ \frac{F(1 - A)\pi r_e^2}{4\pi r_e^2 \sigma} \right]^{\frac{1}{4}} = \left[ \frac{F(1 - A)}{4\sigma} \right]^{\frac{1}{4}},
\]

Where \( F \) is the solar radiation flux in cal/cm\(^2\)-sec, \( A \) is the albedo of fraction of incident radiation reflected, and \( s \) is Boltzmann’s constant. If \( A=0 \) (no energy reflected),

\[
T_{\text{black body}}[\text{°K}] = \left( \frac{0.032 \text{cal/ cm}^2 \text{- sec}\pi r_e^2(1 - A)}{1.35 \times 10^{-12} \text{cal/ cm}^2 \text{- sec- K}^4 4\pi r_e^2} \right)^{\frac{1}{4}} = 277.45 \text{ °K}.
\]

\[
T_{\text{black body}}[\text{°C}] = 277.45 \text{ °K} - 273.15 = 4.3 \text{ °C}.
\]
Example 2: Determine the Sun’s surface temperature from the size of the sun’s disk in the sky and the assumption that its blackbody radiation controls Earth’s surface temperature. The angular diameter of the Sun in the sky is about 0.5° or 8.72x10^-3 radians, so its angular area is that of a disk \( \text{area} = \pi \times \text{radius}^2 \) of radius 4.36x10^-3 radians, i.e. an angular area of 6.05x10^-5 steradians. The total angular area surrounding the Earth is 4\( \pi \) steradians, so the blackbody energy balance determining the surface temperature of the Earth is that the incoming solar radiation energy from the Sun’s disk \( (A_{\text{Sun-disk}})\sigma T_{\text{Sun-disk}}^4 \) is equal to the outgoing terrestrial radiation energy radiated in all directions \( 4\pi \sigma T_{\text{Earth-surface}}^4 \). Balancing these gives \( 6.05 \times 10^{-5} \sigma T_{\text{Sun-disk}}^4 = 4\pi \sigma T_{\text{Earth-surface}}^4 \) or

\[
T_{\text{Sun-disk}} = \left( \frac{4\pi}{6.05 \times 10^{-5}} \right)^{\frac{1}{4}} T_{\text{Earth-surface}} = 21.4 T_{\text{Earth-surface}} \cdot
\]

For a blackbody Earth surface temperature of 277K, this would predict the Sun’s surface temperature to be 5930K, in good agreement with the ~6000K solar surface temperature that is consistent with its yellow color. So we can indirectly determine the Sun’s mass from the Earth’s orbital period, and the Sun’s surface temperature by Earth’s surface temperature.
Problem Set 2c: Black Body Temperature

Problem 1: Given that the flux of heat from the Sun is inversely proportional to the square of the distance from the Sun, use the following table to calculate the black body temperatures of Mercury, Venus and Mars.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance from Sun</th>
<th>Solar Flux</th>
<th>Black Body T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>$58 \times 10^6$ km</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Venus</td>
<td>$108 \times 10^6$ km</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Earth</td>
<td>$150 \times 10^6$ km</td>
<td>1370 J/m^2-sec</td>
<td>$\sim 4.3^\circ C$ ($\sim 277.6$K)</td>
</tr>
<tr>
<td>Mars</td>
<td>$228 \times 10^6$ km</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Problem 2: What is the physical meaning of the black body temperature of a planet?

Problem 3: What would the average black body temperature of the side of the Earth always facing the Sun be if the Earth rotated like the moon, once per orbit? Is your answer reasonable?

Problem 4: The Earth also loses internal energy produced by radioactive decay, with an present-day average surface heatloss of roughly 70 mW/m^2. (Earth’s heatflow varies from $\sim 25$ mW/m^2 to $> 300$ mW/m^2, being lowest in the oldest parts of continents, and highest where two plates are spreading apart.) What fraction of the solar flux is Earth’s internal surface heatloss? Continuing the exploration in the above problem, what would the temperature of the ‘Dark side of the earth’ be if it lost this internal heat while always facing away from the sun? (Neglect any heat being conducted through the Earth from the hot to cold side.)

Problem 5: Over the last 4.5 billion years, the solar heat flux has probably increased by about 20%. Given that the Earth’s black body temperature is now $4.3^\circ C$, what should the Earth’s average temperature have been, if it has increased as the black body law suggests it should?

Problem 6: Temperatures in the Earth’s tropics. Sunlight falls more nearly vertically in Earth’s tropics, thus heating them more than the polar regions. To estimate the appropriate blackbody temperature for Earth’s tropics, assume the equatorial band is a rotating disk in local equilibrium. The tropical belt has area $2\pi r_E W_{EQ}$, where $W_{EQ}$ is the width of the equatorial tropics. Since the tropical belt receives solar energy over an area $2\pi r_E W_{EQ}$ and radiates energy over an area $2\pi r_E W_{EQ}$, what is its predicted blackbody temperature? Does this answer seem reasonable? Why or why not?
Chapter 3: The Age of the Earth and the Geologic Record

Perhaps the most fundamental and venerable question in the Earth Sciences is a very simple one: How old is the Earth and Solar System? Every educated person should know the answer to this question and understand the logic upon which it is based. Radiometric dating techniques provide a remarkably accurate and consistent answer: 4.56 billion years.

The simplest method of dating the Earth is to date the carbonaceous chondrites that accreted and then collected to form the Sun and Earth and other planets of our solar system. Radiometric dating measures the number of daughter atoms created by the decay of a radioactive parent. The more parent atoms, the more daughters produced in a given period of time. Minerals in any natural sample will have different amounts of parent (radioactive) material, and hence will produce different numbers of daughters. For simplicity, we measure the ratio of the present number of parent atoms and the present number of daughter atoms to the number of atoms of a stable isotope of the daughter. For example, if the parent is $^{87}\text{Rb}$, the daughter $^{87}\text{Sr}$, and the stable isotope $^{86}\text{Sr}$, we measure $^{87}\text{Rb} / ^{86}\text{Sr}$ and $^{87}\text{Sr} / ^{86}\text{Sr}$. A key assumption in the radiometric method is the assumption that the initial ratio of $^{87}\text{Sr} / ^{86}\text{Sr}$ has been homogenized by the event that set the radiometric clock. This assumption can be verified by the coherency of the data and appears to almost always be a good assumption. Given at least two measurements of $^{87}\text{Rb} / ^{86}\text{Sr}$ and $^{87}\text{Sr} / ^{86}\text{Sr}$, the dependence of daughter production on parent concentration can be determined and the age of the sample measured.

The essential concept can be appreciated by a simple thought experiment. Imagine that Cornell gets a great new plan to save money by eliminating all clocks in the classrooms, replacing them by a gong that rings every half hour. One drawback to this great cost-saving measure is that there is no obvious way to know how long each class has been going on… until a particularly absent-minded teacher realizes that he doesn’t have to remember, but only needs the class to start by all sitting down, and then have half the remaining sittees stand each time the gong rings, e.g. everyone sits at the beginning of class, half the class stands at the first gong, half the remaining half of people sitting stands at the next gong,... etc. By simply noting if 1, ½, ¼, or 1/8 of the class is still sitting, one can immediately tell if 0, 1, 2, or 3 gongs of time me have passed.

This is the secret of radiometric dating. The sitting people correspond to the radioactive atom, say $^{87}\text{Rb}$. The standing people correspond to the daughter products of radioactive decay. $^{87}\text{Rb}$ decays to $^{87}\text{Sr}$. $^{87}\text{Sr}$ is the ‘daughter’ product. There is one more wrinkle for using the technique to date rocks. There almost always exists a fraction of daughter atoms that either were previously formed by radioactive decay or formed during the creation of the elements within the Solor System. (e.g., in the analogy, some people in the class would have started by standing.) If we had some way to mark the number of people initially standing, the technique would work just fine. In nature, this ‘marking’ is done by taking advantage of the fact that there is usually one or more stable isotopes of any element that is a daughter product of radioactive decay. (e.g $^{86}\text{Sr}$ is a stable isotope of Sr) When a rock melts (the redistribution or homogenizing event) the ratio of $^{87}\text{Sr} / ^{86}\text{Sr}$ is homogenized in all minerals and all samples of the melted material. Homogenization occurs because high temperature chemical reactions do not discriminate between different isotopes of the same element, they just discriminate between different elements. (This occurs because different isotopes have the same atomic size and the same electron bonding structure, while only differing in mass by a percent or so, thus they behave the same in chemical reactions). Typically, different minerals will contain different amounts
of Rb, and different samples of the melt will contain different mixes of minerals and hence different amounts of $^{87}$Rb and different initial ratios of $^{87}$Rb/$^{86}$Sr.

Here is the mathematics describing how this process is used as a clock.

3.1 Radiometric Dating

Let $P$ be the number of radioactive (Parent) atoms in a sample. The probability of decay is proportional to $P$ (we choose a proportionality constant $\lambda$) and time, $t$:

\begin{align}
1) \quad & dP = -\lambda P \, dt \\
& \frac{dP}{P} = -\lambda \, dt \\
& \int_{P_0}^{P} \frac{dP}{P} = \int_{0}^{t} -\lambda \, dt \\
(2a) \quad & \ln \frac{P}{P_0} = -\lambda t
\end{align}

Where $P_0$ is the initial number of parents at $t = 0$.

If $t_{1/2}$ is the time at which $P/P_0 = 1/2$, then:

\begin{align}
2b) \quad & t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.6931}{\lambda} \quad \text{and} \\
(3) \quad & \frac{P}{P_0} = e^{-\lambda t} = e^{-0.6931 t/t_{1/2}} = 2^{-t/t_{1/2}} = \left(\frac{1}{2}\right)^{t/t_{1/2}}
\end{align}

Let $D$ represent the number of atoms that are the product of radioactive decay of a parent atom (called daughter atoms, e.g., $^{87}$Sr). Let $D_0$ be the number of daughter atoms immediately after the homogenizing event at $t = 0$, and $S$ be the number of stable atoms of the same element. Stable here means that $S$ is not the product of radioactive decay. Both $D$ and $S$ are non-radioactive. Then:

$$D = D_0 + P_0 - P.$$
\[ P_0 = \frac{P}{e^{-\lambda t}} = Pe^{\lambda t} \]

\[ D = D_0 + P(e^{\lambda t} - 1), \text{ or} \]

\[ \frac{D}{S} = \frac{D_0}{S} + \frac{P}{S}(e^{\lambda t} - 1) \]

\[ \frac{D}{S} = \frac{D_0}{S} + \frac{P}{S}(e^{0.6931t/t_{1/2}} - 1) \]

\[ \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right]_{\text{now}} = \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right]_{\text{orig}} + \left[ \frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right]_{\text{now}}(e^{\lambda t} - 1) \]

Eqn. 4c is simply the equation for a straight line, with the \( y \)-value of a given point at location \( y = \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right] \) being a function of the \( y \)-intercept or initial \( y_0 = \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right]_{\text{orig}} \), the \( x \)-coordinate of the point \( x = \left[ \frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right] \) and the line’s time-dependent slope \( m = e^{\lambda t} - 1 \). If there are a few minerals that start with different \( x = \left[ \frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right] \) and the same initial \( y_0 = \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right]_{\text{orig}} \), then it is easy to just plot the present-day measured values of \( y = \left[ \frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right] \) for the minerals, read off the \( y \)-intercept from the point where the line through the points crosses the \( y \)-axis, and solve for the age of the rock from the measured slope of the line and the known (e.g. measured elsewhere!) radioactive decay constant for the parent element, i.e. \( t = \frac{\ln(m + 1)}{\lambda} \). This technique works because radioactive decay is a nuclear process that is effectively insensitive to any changes in the chemical and physical environment surrounding the parent atom, i.e. its ‘tick’ is incredibly stable over time.
Problem Set 3: Radiometric Dating

Problem 1: Rewrite (4c) in terms of \((^{87}\text{Rb}/^{86}\text{Sr})_{\text{orig}}\) instead of \((^{87}\text{Rb}/^{86}\text{Sr})_{\text{now}}\).

Problem 2: If the half-life of \(^{87}\text{Rb}\) is 47 Gy, what is \(\lambda\) for \(^{87}\text{Rb}\)?

Problem 3: Whole rock samples collected from the Amitsoq gneiss in the Quilangarssuit area of southwestern Greenland have the \(^{87}\text{Rb}/^{86}\text{Sr}\) and \(^{87}\text{Sr}/^{86}\text{Sr}\) values shown below. The data are plotted below. A line has been “fit” or regressed to the data. Its equation is given at the top of the plot. Note “5.1158e-2x” means 5.128 x 10^{-2}, and “r^2” is the r-squared measure of the “goodness of the fit”, with 1.0 being a perfect fit. What was the initial \(^{87}\text{Sr}/^{86}\text{Sr}\) value? Is this close to the primordial (meteorite) value of .700? Is this reasonable? Why? These gneisses consist of metavolcanic and meta-sedimentary rocks including banded ironstones and conglomerate. What does this mean about conditions on the Earth at the time these samples were metamorphosed and the radioactive clock reset?

\[
\begin{array}{cc}
{^{87}\text{Rb}/^{86}\text{Sr}} & {^{87}\text{Sr}/^{86}\text{Sr}} \\
0.2 & 0.712 \\
0.8 & 0.749 \\
1.0 & 0.754 \\
1.9 & 0.787 \\
2.6 & 0.830 \\
3.25 & 0.874
\end{array}
\]

Amitsoq Gneiss, Quilangarssuit Area Greenland

\[
y = 0.70037 + 5.1158e-2x \quad r^2 = 0.9989
\]
Problem 4: A suite of whole rock samples was collected in the Archean shield (or basement rock older than 2.5 Ga) near the very large Sudbury deposit. Determine the age of the intrusives from the plot.

Whole rock isochron from granites and Gabbros collected near Sudbury, Ontario

Problem 5: The initial ratio of $^{87}\text{Sr}/^{86}\text{Sr}$ differs in the very old Greenland samples and the Sudbury intrusives (problem 4). The Greenland samples have initial values of $^{87}\text{Sr}/^{86}\text{Sr}$ very close to the chondrite value of 0.700. The Sudbury samples have a much higher initial ratio. What can you infer from this observation about the separation of sialic (Rb-loving) material from the mantle?

Problem 6: Samples from a stock (formerly hot igneous intrusive rock mass) in the Archean shield have been collected for Rubidium-Strontium dating. Different minerals were analyzed as indicated below. What is the age of the rock? How long ago did the Archean Eon end? What can you say from your calculation about the intrusive event?

Mineral A: $^{87}\text{Sr}/^{86}\text{Sr} = 0.740$ $^{87}\text{Rb}/^{86}\text{Sr} = 1.18$
Mineral B: $^{87}\text{Sr}/^{86}\text{Sr} = 0.758$ $^{87}\text{Rb}/^{86}\text{Sr} = 1.73$
Chapter 4: Geology - The Story of the Earth’s Last 4.56 Ga.

For the first 1.5 Gy (billion years) or so since 4.56 Ga, the Earth was intensely bombarde by meteorites and comets. During this time it lost essentially all of its atmosphere at least once, during the moon-forming impact. (In contrast, moonless Venus retained much of her original heavy volatile atmophile elements such as the rare gas Argon). The record preserved on the moon’s surface shows that the period of heavy bombardment ended at roughly ~3.9Ga

The Earth quickly achieved a state similar to the one it now has. The oldest known rocks on Earth are 3.96 billion years old and are sediments with volcanic fragments found in Canada. The existence of sediments this old means that since very early times there was flowing liquid water on the Earth’s surface. (The oldest zircon mineral so far found in sediments is a 4.2Ga zircon from Australia. In striking contrast, of the 7 near-random points on the moon from which rock samples have been collected and brought back to Earth to be dated, rocks as old as 4.3Ga have been found.) Furthermore, over most of the last 83% of the Earth’s 4.56 billion year history the temperature of the Earth has been within a narrow band where the oceans have neither frozen nor boiled. From isotopic and fossil evidence, the temperature of the Earth has been within a few tens of degrees of its present values for most of its ~4 billion years. If anything, the Earth has cooled slightly with time despite a 10 to 20% increase in the radiancy (watts of radiant energy emitted per m$^2$ of the solar surface) of the Sun. The exception to this generalization may have been the late Proterozoic when the oceans did freeze completely about 4 times and the Earth oscillated between super-icehouse (-50°C) and super greenhouse (+50°C) conditions. This is the “snowball Earth” hypothesis of Kirschvink, recently championed by Hoffman and Schrag. These drastic temperature variations posed a dramatic challenge to early life. They may have forced the development of super-adaptable species, and prepared the way for the explosion of different life forms in the Phanerozoic eon that followed.

Sedimentation has operated on Earth for the last 4 Ga. So what? The fact is sediments provide a key window into the Earth’s past. Herodotus, a Greek philosopher, was one of the first to realize the potential of sediments for reaching into the distant past. In 450 BC he discussed the shape of the Nile delta, and commented that such a large pile of sediments must have taken a very long time to accumulate from the few centimeters deposited each year during the annual flooding of the Nile. Herodotus' characterization of the form of the river sedimentation as a Greek Δ has continued to this day in the word river "delta". Sedimentary basins, the areas where sediments accumulate, are the Earth’s garbage cans, areas where the products of erosion, dead animals, shells and bones are deposited in a time-ordered stack with the oldest units on the bottom and the youngest on top. Geologic history, like human archeology, is told largely in terms of who has died and what nature has thrown away. The study of fossil remains in sedimentary sequences, now dated by the radiometric methods discussed in Chapter 3, provides a fascinating view of how life evolved on the Earth. Changes in evolution provided the initial basis for dividing geologic time into periods. Even before it was realized that fossils are remains of dead animals, the appearances and disappearances of different fossil types (e.g. trilobites, dinosaurs) was being used to mark relative ages.

In working out a geologic time scale, geologists did just what anyone would do. They put boundaries where there were recognizable changes in fossils (Figure 4.1). All the geologic boundaries are marked by distinctive evolutionary changes. The division of the geologic time scale thus tells us about extinctions, and the evolutionary discontinuities caused by the extinctions. The most important and longest divisions are
called "eons" and "eras". The next most important are "periods", and the finest divisions are "epochs".

Although early in the Earth’s history there were no hard-shelled fossils, microfossils of early life forms can be detected under a microscope. From such evidence we know that life probably started about 4 billion years ago. Bacteria and algae were well established by 3.8 Ga (billion years ago). By about 2.5 Ga they were well enough established that they produced, through photosynthesis, an oxygen-rich atmosphere similar to the one we have today. The oxygen was released as a by-product or waste that those organisms disposed of. The interaction of that “poisonous” (or oxygenating) atmosphere with the barren rocks of the continents leached iron in massive quantities which accumulated in almost pure form in shallow seas, forming (about 2.5 Ga) what still remain the Earth’s richest, most extensive, and most valuable iron deposits. The Mesabi Range in Michigan is one of these Superior-type sedimentary iron deposits. Early or Precambrian time is divided into the Archean Eon (or pre-oxygenating atmosphere, to 2.5 Ga) and Proterozoic Eon (post oxygenating atmosphere). The eon of shelled fauna (from the time they developed to present) is called the Phanerozoic Eon. The Phanerozoic Eon is divided into the Paleozoic, Mesozoic, and Cenozoic Eras.

The first shells appear in the first period of the Paleozoic Era. The Cambrian Period ran from the end of the Proterozoic at 544 Ma to the Ordovician Period, which started 505 million years before present. Shortly before the Cambrian, there were soft bodied worms and jellyfish-like creatures called Ediacaran fauna. Marine invertebrates (graptolites, trilobites) dominate the fossil record of the Ordovician, which lasted from the end of the Cambrian until 440 million years ago. Fish appeared in the Silurian Period that lasted from 440 to 410 Ma. Land plants and trees evolved at the end of the Devonian (why not sooner?), reptiles at the end of the Carboniferous, and the mega-continent Pangea was assembled in the Permian (286-251 Ma). The time from Cambrian to Permian is called the Paleozoic Era. The names of the Paleozoic periods can be remembered with the phrase "Come Over Some Day Maybe Play Poker" for Cambrian, Ordovician, Silurian, Devonian, Mississippian, Pennsylvanian and Permian. In the table below the Carboniferous period includes both the Mississippian and Pennsylvanian periods. The end of the Permian marks the start of the Mesozoic Era. The Mesozoic Periods (Triassic, Jurassic, and Cretaceous) saw the peak development of the dinosaurs and the evolution of mammals. Almost all species preserved in the fossil record are now extinct; only 0.1% of all species that have lived on Earth live on present-day Earth.
A major change occurred at the end of the Mesozoic Era, at the Mesozoic-Cenozoic boundary, or as it is often called, the Cretaceous-Tertiary boundary. The Tertiary is the first Cenozoic Period. At this time, ~66 million years ago, the dinosaurs "suddenly" became extinct, and small, hairy, warm-blooded mammals that had been evolving in coexistence with dinosaurs for over ~100Ma inherited the Earth, replacing dinosaurs as the largest grazing animals and predators. Initially there was much debate about the cause or causes of this important extinction event (and evolutionary opportunity). Some believed the extinctions at the Cretaceous-Tertiary (or K-T) boundary were caused by a large meteorite hitting the Earth. Others believed the cause was particularly intense volcanic eruption (the outpouring of the Deccan Traps or flood basaltic lavas in India) associated with normal plate tectonic processes. The debate still rumbles, but has been settled for most with the discovery of the Chixalub impact site off the Yucatan Peninsula.

Not all extinction events may be due to impacts. In fact this cause may be relatively rare. For example, in the largest mass extinction ever that occurred at the Permian-Triassic boundary at 251 Ma (million years), the species that died off were almost all marine invertebrates, and the extinctions occurred in at least two major pulses (251Ma and 257Ma), not a single event as the meteorite impact hypothesis would suggest. There is also ongoing debate as to how long-lived each extinction pulse was. The problem is that the geologic record is ‘scratched’, and hard to read at the detail necessary to answer these questions. There is a beautiful exhibit covering this extinction in the display case on the ground floor of Snee, and in the Hall of Life at PRI across the lake.
The Cenozoic Era is divided into the Tertiary and Quaternary Periods. The Tertiary Period is divided into the Paleocene, Eocene, Oligocene, Miocene, and Pliocene Epochs, and the final Quaternary Period is divided into the Pleistocene (or glacial epoch) and the Holocene (or post-glacial epoch), which started 10,000 years ago.

The span of geologic time is difficult to grasp intuitively. By $\sim 2.5 \text{ Ga}$ (billion years) life had developed sufficiently to produce an oxygenating atmosphere. It took until $\sim 0.57 \text{ Ga}$ (the start of the Cambrian era) for creatures with skeletons to develop. From then on changes were rapid. Species tended to evolve from small to all sizes including very large, and then, at major extinction events, back to small. The first hominoids made their appearance about 0.005 Ga ago, and recorded human history began about 4000 BC or 0.000006 Ga ago. If the 4.56 Ga evolution of the Earth is taken as one day running from 12 midnight to 12 midnight, the Cambrian era started at 8:58 p.m. (3 hours ago), the dinosaurs became extinct 21 minutes ago, the first hominids appeared 1 minute before midnight, and recorded history started one tenth of one second ago.

Clearly, a historical record of 0.1 second is not much of a basis for projecting what the past day was like. The litany of all the disasters and favorable conditions that have affected humans is not much of a basis for projecting what the Earth has been like or will become. To address such questions we must extend our horizons by using our intelligence and deductive powers to infer what has happened on Earth before we were able to keep written records. We must study geology, the Earth’s own written record. Sherlock Holmes was a great detective because he combined acute powers of observation with superb intellect. A good Earth scientist, like a good detective, must combine both these skills. Either one alone is insufficient. For today's Earth scientist, intellect must include quantitative reasoning. Simple logic is no longer adequate.

**Additional Reading**