Chapter 7
Deformation and Infinitesimal Strain

Introduction

In this chapter, we turn our focus to deformation — the quantification of changes in shape and/or volume of a rock, plus any associated translations and rotations — that is the bread and butter of most structural geologists. It will be a significant departure from the topics of the last two chapters in several fundamental ways: stress is an instantaneous property that exists only in the instance that the force is applied. Deformation, on the other hand, still exists in rocks hundreds of millions, or even billions, of years after the associated stresses have dissipated. Deformation is also cumulative: one can superpose many episodes of deformation produced by completely unrelated geologic events. Finally, because deformation represents a change in shape or size, it is a comparison between two different states, an initial and a final state of the material. Eventually, we will have to come to grips with the fact that these represent two possible reference states. Whenever we consider the change in some property, we are really talking about mathematical derivatives: local slopes along a curve of a continuous function. In our case, we will be looking at the change in position, or the change in displacement, with respect to position.
Strain

One Dimensional Measures of Strain

A structural geologist can make three different types of measurements in order to quantify strain: changes in line length, angles, or volumes. In the discussion that follows, we’ll use a capital “X” to indicate the coordinate of a point in the initial state (the material coordinate system) and a small “x” for the final state (the spatial coordinates, implicitly at time, t). We’ll start with the change in length of a line (Fig. 7.1). As you can see in Figure 7.1, the initial and final lengths of the line are:

\[
\ell_i = X_b - X_a = \Delta X \quad \text{and} \quad \ell_f = x_b - x_a = \Delta x
\] (7.1)

Thus, the stretch, S can be defined as:

\[
S = \frac{\ell_f}{\ell_i} = \frac{\Delta x}{\Delta X}
\] (7.2a)

\[
s = \frac{\ell_i}{\ell_f} = \frac{\Delta X}{\Delta x}
\] (7.2b)

The initial length, \(\ell_i\), in the denominator of Equation (7.2a) means that the initial state is the reference state. In (7.2b), \(\ell_f\) occurs in the denominator so the final state is the frame of reference. Another way to describe this deformation is by looking at the displacement, \(u\), of the end points of the lines:

\[
\begin{align*}
X_i & \quad X_f \\
u_a & \quad u_b
\end{align*}
\]

Figure 7.1 — The change in length of a line. The initial state is shown in blue with capital “X”s and the final state in red with small “x”s. \(u_a\) and \(u_b\) are the displacements of the end points of the line.
And the change in displacement, \( \Delta u \), is:

\[
\Delta u = u_b - u_a = (x_b - X_b) - (x_a - X_a) = (x_b - x_a) - (X_b - X_a) = \ell_f - \ell_i = \Delta \ell
\]  \hspace{1cm} (7.4)

Thus, the extension, \( E \) or \( e \), can be defined as the change in length over the initial length (an initial state frame of reference) or the final length (a final state frame of reference):

\[
E = \frac{\ell_f - \ell_i}{\ell_i} = \frac{\Delta \ell}{\ell_i} = \frac{\Delta u}{\Delta X} \hspace{1cm} (7.5a)
\]

\[
e = \frac{\ell_f - \ell_i}{\ell_f} = \frac{\Delta \ell}{\ell_f} = \frac{\Delta u}{\Delta x} \hspace{1cm} (7.5b)
\]

Now let’s turn our attention to changes in angles and to do so we need to bring a second dimension which we will (temporarily) call \( Y \) and \( y \). Consider a line initially perpendicular to the X direction that is displaced in the X direction by an amount that varies with the distance, or length, \( \Delta Y \) (Fig. 7.2). We can write:

\[
\begin{align*}
    u_a &= x_a - X_a \quad \text{and} \quad u_b = x_b - X_b \\
    \Delta u &= (x_b - X_b) - (x_a - X_a) = (x_b - x_a) - (X_b - X_a)
\end{align*}
\]  \hspace{1cm} (7.6)

But, \( (X_b - X_a) = 0 \), so

\[
\Delta u = (x_b - x_a)
\]

We define the shear strain, \( \gamma \), and the angular shear, \( \psi \), as:
\[ \gamma = \frac{\Delta u}{\Delta Y} \quad \text{and} \quad \psi = \tan^{-1}\left(\frac{\Delta u}{\Delta Y}\right) \quad (7.7) \]

As we will see a bit later, \( \gamma \) as defined in Equation 7.7 is actually known as the \textit{engineering shear strain} to distinguish it from a similar but distinct quantity known as the \textit{tensor shear strain}.

\textit{Three Dimensional Deformation}

Now it is time to extend these concepts to three dimensions. The ratios in Equations (7.2) and (7.5) are gradients of change in position with respect to the new or old position and gradients of displacement with respect to position. In an arbitrarily deformed body, these should vary in the three directions of our Cartesian coordinate systems. That is:

\[ \frac{\Delta x}{\Delta X} = \lim \frac{\partial x_i}{\partial X_j} = D_{ij} \quad \text{and} \quad \frac{\Delta u}{\Delta X} = \lim \frac{\partial u_i}{\partial X_j} = E_{ij} \quad (7.8) \]

where \( D_{ij} \) is the \textit{deformation gradient tensor} and \( E_{ij} \) is the \textit{displacement gradient tensor}. The equations in (7.8) are referenced to the initial state and, as you might expect, there are equivalent forms referenced to the final state. We need to use partial derivatives because the displacement is a function of gradients along the three axes of the coordinate system.

Figure 7.3 — The deformation of line PQ in undeformed state (in blue) to P’Q’ in the final state (in red).
Figure 7.3 shows a more general case. The difference in displacement vectors is:

\[ \Delta u_i = \vartheta u_i - \phi u_i = \frac{\partial u_i}{\partial X_j} \vartheta X_j - \frac{\partial u_i}{\partial X_j} \phi X_j = \frac{\partial u_i}{\partial X_j} (\vartheta X_j - \phi X_j) = \frac{\partial u_i}{\partial X_j} \Delta X_j \]  \hspace{1cm} (7.9)

Thus, we can write:

\[ \Delta u_i = \frac{\partial u_i}{\partial X_j} \Delta X_j = E_{ij} \Delta X_j \]  \hspace{1cm} \text{or} \hspace{1cm} du_i = \frac{\partial u_i}{\partial X_j} dX_j = E_{ij} dX_j \hspace{1cm} (7.10)

We can integrate the right side of equation (7.10) to yield a general expression for the displacement at any position:

\[ \int du_i = \int E_{ij} dX_j \ \Rightarrow \ \ u_i = t_i + E_{ij} X_j \]  \hspace{1cm} (7.11)

where \( \mathbf{u} \) is the displacement vector at position \( \mathbf{X} \), and \( \mathbf{t} \) is a constant of integration that represents the displacement of a point at the origin of the coordinate system. This equation holds as long as the strain is \textit{homogeneous}; that is, \( E_{ij} \) is the same throughout the deformed body. Because Equation (7.11) represents three linear equations, it follows that \textit{a line that is straight before deformation will also be straight after the deformation}; likewise \textit{parallel lines in the initial state will remain parallel in the final state}.

Just as we did for the displacement gradient tensor, the relationships depicted in Figure 7.3 can also be used to derive an expression for mapping points in the initial state into the final state:

\[ \Delta x_i = \vartheta x_i - \phi x_i = \frac{\partial x_i}{\partial X_j} \vartheta X_j - \frac{\partial x_i}{\partial X_j} \phi X_j = \frac{\partial x_i}{\partial X_j} (\vartheta X_j - \phi X_j) = \frac{\partial x_i}{\partial X_j} \Delta X_j = D_{ij} \Delta X_j \]  \hspace{1cm} (7.12)

Once again, assuming homogeneous strain, we can integrate both sides of the equation to get:

\[ \int dx_i = \int D_{ij} dX_j \ \Rightarrow \ x_i = c_i + D_{ij} X_j \]  \hspace{1cm} (7.13)
Where \( x \) is the new position, \( X \) is the old position, and \( c \) is a constant of integration that represents the coordinates of a point initially at the origin of the coordinate system.

Recall that a second order tensor is a linear vector operator. The displacement gradient tensor relates the displacement of a point to its position, whereas the deformation gradient tensor relates the position in the initial state to the position of the same point in the final state (Fig. 7.4). If we know the tensor, then the displacement of the point and its new position can be calculated from Equations (7.11) and (7.13). All of the equations that we have developed so far hold for any magnitude of deformation. To understand the nature of these tensors, and explore some common applications, some simplifying assumptions are in order.

**Infinitesimal Strain**

If we assume that the distortions are small, a number of simplifications can be made. At the most basic level, a small or *infinitesimal strain* assumption permits us to consider that the initial and final states are identical, thus cutting in half the number of tensors to worry about. Additional benefits will become apparent,
below, but first let’s take a closer look at the components of the displacement gradient tensor, $E_{ij}$.

It will probably come as no surprise the values along the principal diagonal, $E_{11}$, $E_{22}$, and $E_{33}$ represent the extensions of lines that are parallel to the corresponding axes. The off-diagonal components are more interesting, which will be illustrated by examining a special case in two dimensions (Fig. 7.5). From the geometry, you can see that:

$$\tan \theta = \frac{\Delta u_2}{\Delta X_1 + \Delta u_1} \tag{7.14}$$

and by making our infinitesimal strain assumption, you can see that $\Delta X_1 \gg \Delta u_1$. Thus, we can write that:

$$\tan \theta \approx \frac{\Delta u_2}{\Delta X_1} \tag{7.15}$$

For very small angles, the tangent of an angle is equal to the angle itself measured in radians, so we can further write:

$$\theta \approx \frac{\Delta u_2}{\Delta X_1} = \frac{\partial u_2}{\partial X_1} = E_{21} \tag{7.16}$$

Thus $E_{21}$ is the counterclockwise rotation of a line parallel to the $X_1$ axis towards the $X_2$ axis. Likewise, $E_{12}$ would be the clockwise rotation of a line parallel to the $X_2$ axis towards the $X_1$ axis. The first subscript indicates the axis that the rotation is
towards and the second subscript indicates the axis to which the line is, initially, parallel. Note that the $E_{21} + E_{12}$ combined equal the change in angle, or shear strain, of two lines initially at $90^\circ$ to each other. They are tensor shear strain components and each is equal to one-half of the engineering shear strain of Equation (7.7).

There is one more thing to learn about the displacement gradient tensor and, by extension, the deformation gradient tensor as well. Again, we’ll examine a special case (Fig. 7.6). As in Figure 7.5, there are two vectors in the initial, undeformed configuration that are parallel to the coordinate axes. This time, however, we introduce a pure rotation by a small angle, $\phi$, with no deformation; that is, the vectors $P'Q'$ and $P'M'$ are still perpendicular after the rotation. You can see that:

$$E_{11} = \frac{\Delta u_1}{\Delta X_1} = 0 \quad \text{and} \quad E_{21} = \frac{\Delta u_2}{\Delta X_1} = \tan \phi = \phi$$

Likewise, $E_{22} = 0$ but $E_{12} = -\phi$ because it is also a counterclockwise rotation even though positive $E_{12}$ should be a clockwise rotation. Thus our two dimensional displacement gradient tensor for the case of pure rotation is:

$$E_\phi = \begin{bmatrix} 0 & -\phi \\ \phi & 0 \end{bmatrix}$$

We’ve just learned two really important things: First, the displacement gradient tensor is an asymmetric tensor (because $-\phi \neq \phi$), and second, the tensor includes both strain and rotation.
Any asymmetric matrix can be additively decomposed into a symmetric matrix and an antisymmetric matrix. An antisymmetric matrix has zeros along the principal diagonal and the values below the principal diagonal must be the negative of those above the principal diagonal. For the displacement gradient tensor, we can write:

\[ E_y = \varepsilon_y + \omega_y \]  

(7.19)

where

\[ \varepsilon_y = \frac{1}{2}(E_y + E_{ij}) \quad \text{and} \quad \omega_y = \frac{1}{2}(E_y - E_{ij}) \]  

(7.20)

\( \varepsilon_y \) is the symmetric infinitesimal strain tensor and \( \omega_y \) is the antisymmetric rotation tensor or axial vector. \( \omega_y \) can be turned into a rotation vector, \( r_i \), as follows:

\[ r_1 = \frac{-(\omega_{23} - \omega_{32})}{2}, \quad r_2 = \frac{-(\omega_{31} + \omega_{13})}{2}, \quad \text{and} \quad r_3 = \frac{-(\omega_{12} - \omega_{21})}{2} \]  

(7.21)

the magnitude of \( r \) gives the amount of rotation (in radians) and the unit vector, \( \hat{r} \), gives the orientation of the rotation axis. Equation (7.19) basically says that deformation is equal to a strain plus a rotation, thought that is strictly true only for small strains.

Like any symmetric tensor, the infinitesimal strain tensor has principal axes, found by solving the eigenvalue problem, and invariants. The first invariant of the infinitesimal strain tensor:

\[ I_\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]  

(7.22)

is the infinitesimal volume strain or the dilatation. The infinitesimal strain ellipsoid is defined by the equation:

\[ \frac{x_1^2}{(1+\varepsilon_1)^2} + \frac{x_2^2}{(1+\varepsilon_2)^2} + \frac{x_3^2}{(1+\varepsilon_3)^2} = \frac{x_1^2}{S_1^2} + \frac{x_2^2}{S_2^2} + \frac{x_3^2}{S_3^2} = \frac{x_1^2}{\lambda_1} + \frac{x_2^2}{\lambda_2} + \frac{x_3^2}{\lambda_3} = 1 \]  

(7.23)
where \( S_1, S_2, \) and \( S_3 \) are the stretches along the principal axes and \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the quadratic elongations. If the intermediate principal stretch is 1, the deformation is two dimensional and we refer to it as **plane strain**.

Just like any symmetric tensor, the infinitesimal strain tensor can be represented by a Mohr’s Circle construction (Fig. 7.7). We start with the axes of the infinitesimal strain ellipse parallel to the axes of the coordinate system:

\[
\varepsilon_{ij} = \begin{bmatrix}
\varepsilon_1 & 0 & 0 \\
0 & \varepsilon_2 & 0 \\
0 & 0 & \varepsilon_3 \\
\end{bmatrix}
\]  

(7.24)

And transform the tensor by a rotation of \( \theta \) about the intermediate principal axis (Fig. 7.7a). The transformation matrix is:

\[
a_{ij} = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta \\
\end{bmatrix}
\]  

(7.25)

The tensor transformation equation is:
So, the infinitesimal strain tensor in the new coordinate system is:

\[
\varepsilon'_{ij} = a_ia_j\varepsilon_{kl}
\]  

(7.26)

And the equations for Mohr’s Circle for infinitesimal strain (Fig. 7.7b) are:

\[
\varepsilon'_{11} = \frac{(\varepsilon_1 + \varepsilon_3)}{2} + \frac{(\varepsilon_1 - \varepsilon_3)}{2}\cos2\theta
\]

\[
\varepsilon'_{13} = \frac{\gamma}{2} = \frac{(\varepsilon_1 - \varepsilon_3)}{2}\sin2\theta
\]

(7.28)

One of the most important features of infinitesimal strain, which is made especially clear by the Mohr’s Circle construction, is that the maximum infinitesimal shear strain is oriented at 45° to the principal axes of strain.

This has profound implications for our practical study of common structures, from brittle to ductile shear zones (Fig. 7.8).

Figure 7.8 — Three geological/geophysical examples demonstrating the importance of the fact that the principal axes of infinitesimal strain are at 45° to the planes of maximum shear strain. (a) A heterogeneous ductile shear zone in granitoid rocks, (b) sigmoidal extension fractures in a brittle shear zone, (c) P & T axes for an earthquake or fault slip analysis. Despite their names, P and T axes are infinitesimal principal strain axes.
Some Geological Applications

Many structural geology problems we face break the assumptions of infinitesimal strain and thus require a more complicated analysis. However, the fields of active tectonics and brittle fault analyses are of significant importance for modern structural geology and are amenable to an infinitesimal strain approach. That is because the deformation accrues over a short, commonly geologically instantaneous, period of time and thus is very small in magnitude.

**Strain from GPS**

The Global Positioning System (GPS) has revolutionized earth sciences in the last 25 years by providing geologists and geophysicists with real time monitoring of active deformation. Modern continuous geodetic GPS provides sub-centimeter resolution of the displacement of monuments or stations relative to a stable reference frame. Because the changes in displacements measured are on the order of centimeters between stations separated by tens of kilometers, the deformation measured by GPS certainly qualifies as infinitesimal. GPS data present a common circumstance: we know displacements and want to calculate the strain rather than knowing the strain tensor and calculating displacements at different points.

We start with a simpler task: calculating the one-dimensional extension given a transect of GPS stations. Recall that, from Equation (7.7), the one dimensional extension is just:

\[ E = \frac{\Delta u}{\Delta X} \]

![Figure 7.9 — Plotting displacement against position in a GPS transect in order to determine the 1D extension. Because the slope is positive, the data represent an elongation; a negative slope would indicate shortening in the direction of the transect.](image-url)
Therefore, one can plot the displacement at a station on the Y-axis against the position of the station on the X-axis and fit a straight line (for homogeneous strain) or a curve (for heterogeneous strain) to the data points as in Figure 7.9. The equation for a straight line on the graph is:

$$u = t + EX$$

(7.29)

where $t$ is the intercept on the vertical axis. Note how similar this equation is to Equation (7.11).

To get to the point of making a graph like that shown in Figure 7.9 you will probably have to do several intermediate steps, which we have the background to do. It is unlikely that the GPS displacement (or velocity) vectors, or the transect, will be parallel to North or East. While you could eyeball the best orientation of the transect and then calculate the distances between each station, there is a more elegant way. First, calculate the orientation of the mean GPS vector just like we did in Chapter 2. Then rotate the coordinate system so it is parallel to the mean vector and transform both the station positions and the displacement vectors into this new coordinate system (Fig. 7.10). The two dimensional transformation matrix for this operation will be:

$$a_{ij} = \begin{pmatrix} \cos \theta & \cos(90 - \theta) \\ \cos(90 + \theta) & \cos \theta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

(7.30)
And the vector and station transformations will be, respectively:

\[ u'_i = a_{ij}u_j \quad \text{and} \quad X'_i = a_{ij}X_j \]

(7.31)

Now, you can plot the component of the displacement parallel to \( X'_i, u'_i \), against the station’s \( X'_i \) coordinate (ignoring the \( X'_2 \) components completely), as in Figure 7.9. Thus, expanding Equation (7.31) for those components only, we get:

\[ u'_i = a_{11}u_1 + a_{12}u_2 \quad \text{and} \quad X'_i = a_{11}X_1 + a_{12}X_2 \]

(7.32)

To get the two (or three) dimensional strain from displacement vectors requires more involved methods. Recall that the multidimensional equivalent of Equation (7.29) is equation (7.11) which is repeated here:

\[ u_i = t_i + E_{ij}X_j \]

We know the displacement vectors, \( \mathbf{u} \), and the positions of the stations, \( \mathbf{X} \). We don’t know \( E_{ij} \) or \( t_i \); in two dimensions (ignoring the vertical component of the GPS data), there are thus six unknowns: \( E_{11}, E_{12}, E_{21}, E_{22}, t_1, \) and \( t_2 \). Each station and displacement vector pair provides two equations (one for \( u_1 \) and one for \( u_2 \)), so in two dimensions, we need three non-collinear stations and displacements to solve for our six unknowns. You can visualize the three stations defining a triangle with a circle inscribed in it; when the points are displaced the triangle becomes distorted and the inscribed circle becomes a strain ellipse (Fig. 7.11).

To calculate the unknowns, we need to gather them into a single matrix:

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**Figure 7.11** — In 2D, three stations and displacement vectors are necessary to define the strain produced by the displacements. The strain here is finite (i.e., large), but the same principle holds for infinitesimal deformations like that measured by GPS.
This equation is written, not for three stations but for \( n \) stations and vectors — more information than we actually need. That’s okay, though, because with data redundancy, we can calculate the uncertainties using a least squares approach. Solving Equation (7.33) is non-trivial because it requires calculating the \textit{inverse} of the large matrix full of zeros, ones, and X’s. The techniques required to do this — generally referred to as \textit{inverse methods} — are extremely powerful and form the basis of many very important calculations in earth sciences and other fields but are beyond the scope of this book. If you want to be a modern earth scientist, you will probably eventually want to learn these methods.

\textit{Analyzing Brittle Faults}

The upper crust of the earth is full of faults — discontinuous deformation features. If we assume that the faults are small relative to the volume of earth that contains them, we can analyze them using infinitesimal strain assumptions. The simplest approach is to leverage the fact that the plane of maximum shear strain is at 45° to the principal axes of infinitesimal strain. In fault slip (and earthquake) analysis, the principal infinitesimal shortening axis is referred to by the letter “P” and the principal elongation axis with the letter “T” (Fig. 7.8c). Faults are particularly easy to analyze because there is no movement perpendicular to the slip vector and thus each individual fault represents deformation in plane strain. The basic geometry is shown in Figure 7.12.

If one is only interested in calculating the orientations of the individual P and T axes, the calculation is quite straightforward, involving nothing more than
vector addition and subtraction. Figure 7.13 is a view perpendicular to the movement plane so that one sees the fault plane edge on and it appears as a line. The unit pole vector, \( \hat{n} \), points down into the footwall and the unit slip vector, \( \hat{s} \), also points down into the lower hemisphere (i.e., we are using a NED coordinate system). We need some way to specify the sense of slip, which we will do with a scalar value, \( k \): for faults with a normal component of slip the hanging wall moves down and \( k = 1 \); for those with a reverse component, \( k = -1 \), because the hanging wall moves up whereas down in positive. Thus, we can write:
The square root of two in the denominator is necessary to ensure that \( \mathbf{P} \) and \( \mathbf{T} \) are unit vectors.

The preceding analysis is fine if we are concerned about calculating \( \mathbf{P} \) and \( \mathbf{T} \) axes of single faults, but what if we want to calculate the strain produced by a group of faults? For this case, we introduce a new concept from earthquake seismology, the scalar \textit{seismic moment}, \( M_o \), and it’s sibling, the \textit{geometric moment}, \( M_g \):

\[
M_o = A\bar{\tau}\mu \quad \text{and} \quad M_g = A\bar{\tau}
\]  

(7.35)

where \( A \) is the surface area that slipped, \( \bar{\tau} \) is the average displacement, and \( \mu \) is the shear modulus. The moment tensor is the sum of the scalar moment times the dyad product of the unit slip and normal vectors:

\[
M_{ij} = \sum_{1}^{n\text{ faults}} M_g u_i n_j
\]

(7.36)

where \( u_i \) is the unit slip vector and \( n_j \) the unit normal (or pole) vector. This equation is for the geometric moment tensor. If you calculated \( u_i n_j \) for a single fault and calculated the eigenvalues and eigenvectors, it would give us the \( \mathbf{P} \) and \( \mathbf{T} \) axes, just like Equation (7.34). It turns out that our old friend, the displacement gradient tensor, is equal to (Molnar, 1983):

\[
E_{ij} = \frac{\sum_{1}^{n\text{ faults}} M_g u_i n_j}{V} = \frac{M_g}{V}
\]

(7.37)

The summation of moment tensors of all of the faults in the data set is a simplification afforded us by the infinitesimal strain assumption. When the faults are large enough to cause finite strain, you can no longer add the tensors together. Instead, matrix multiplication is involved as we will see in a subsequent chapter.
CHAPTER 7

INFINITESIMAL STRAIN

\( E_{ij} \) of Equation (7.37) is, of course, an asymmetric tensor and in infinitesimal strain can be additively decomposed into a symmetric strain and antisymmetric rotation tensors:

\[
E_{ij} = \varepsilon_{ij} + \omega_{ij} = \frac{1}{2V} \sum M_u (u_{nj} + u_{nj}) + \frac{1}{2V} \sum M_u (u_{nj} - u_{nj})
\]

(7.38)

The symmetric part of (7.38), before being divided by \( 2V \) is Kostrov’s (1974) symmetric seismic moment tensor. When you see a moment tensor reported in an earthquake catalog, it is Kostrov’s symmetric tensor and the infinitesimal strain, \( \varepsilon_{ij} \) in that case is:

\[
\varepsilon_{ij} = \frac{1}{2V \mu} \sum M_u (u_{nj} + u_{nj})
\]

(7.39)

If you don’t have any scaling information — that is, you don’t know \( M_o, M_g, V, \) or \( \mu \) — you can still add up the moment tensors and get the principal axes of the group of faults assuming that each fault contributes equally to the overall strain in the region. If, however, some faults (or earthquakes) are markedly larger than others, that is a poor assumption; the moment tensor sum is always dominated by the largest features.
Exercises—Chapter 7

All of the following exercises should be done either in a spreadsheet or Matlab. You will need the EigenCalc program that you downloaded last week for some of the exercises. All of the exercises have datasets that can be downloaded from the course web page.

1. Calculate the 1D coseismic strain for the 1995 Antofagasta earthquake, using the data from Klotz et al. (1999). The data will be provided to you in a spreadsheet. Carry out the following tasks:

(a) Determine the average or mean vector that characterizes, as best possible, the overall orientation of the vectors.

(b) Determine the two-dimensional transformation matrix, $a_{ij}$, needed for a new coordinate system where the $X'_1$ axis is parallel to the mean vector direction.

(c) Transform the East and North coordinates of the GPS stations, vectors, and errors into the new coordinate system.

(d) Plot the $u'_1$ component of each displacement vector, and its error, against the $X'_1$ component of the station position.

(e) Fit a straight line to approximately linear segments of the resulting curve using the relations or built in functions from the previous sections.

2. Fifteen measurements of faults and their slickensides are given in the table below. Calculate the $P$ and $T$ axes of the individual faults and then calculate an unweighted moment tensor summation. Use EigenCalc to determine the principal axes of infinitesimal strain. In the downloadable spreadsheet, the sense of slip (SOS) has been specified using $k=+1$ for normal faults and $k=-1$ for reverse faults as described in the text (Eqn. 7.34).

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<td>317.1</td>
<td>31.6</td>
</tr>
<tr>
<td>93.7</td>
<td>65</td>
<td>S</td>
<td>269.6</td>
<td>8.8</td>
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<td>297.6</td>
<td>64.1</td>
<td>N</td>
<td>300.2</td>
<td>5.4</td>
</tr>
</tbody>
</table>
3. You’ll be given a spreadsheet containing 57 foreshocks of the Mw 8.2 Pisagua earthquake (northern Chile March-April 2014) which have nodal planes and scalar moments.

(a) Calculate the moment tensor sum for the foreshocks and use EigenCalc to determine the orientation of the principal axes.

(b) Plot the P and T axes of the individual events in Stereonet and compare them to the moment tensor sum that you did in part (a).