**Thermal Subsidence**

**Thermal Properties/Cooling of Lithosphere**

Thermal contraction/subsidence is one of the fundamental types of tectonic subsidence. In other words, thermal perturbations are one of the key ways in which isostatic equilibrium is disturbed, thus resulting in vertical motions that re-compensate.

General Setting of a Thermal Disturbance:

Asthenosphere, which is comparatively mobile, moves upward to depths that are normally those of the lithosphere, and carries with it HEAT. (Heat is conveceted). Because it can move relatively fast, the heat is not conducted nor convected away into the neighboring rocks rapidly enough to maintain horizontal isotherms (ie, this is adiabatic). Thus the result is a hotter-than-average blob at lithospheric depths.

- "Thermal" (2) and an instantaneous "isostatic" (1) effects:
  
  1. If the asthenosphere is replacing lower density rock (ie, crust rather than mantle), then the mass of the rocks in the column has increased and the surface must subside (to cause more air or water or sediment, of low density) to compensate.

  2. The asthenospheric blob will cool off by transferring heat to once-cold rocks (conduction). Heating those rocks causes them to reduce in density/increase in volume. Thus they occupy more space, causing them to rise (the surface goes up, rather than down). Simultaneously, the heat lost by the asthenosphere allows it to contract (density increases, volume decreases).

The first response is instantaneous, or at least matches the time scale of the deformation/intrusion that allowed the hot material to enter the shallow level. The second, the heat loss and concomitant thermal expansion/contraction, is a long term process.
Under most circumstances, for rifting of continental lithosphere, subsidence dominates uplift, in other words, the isostatic response to replacing low density crust by high density mantle, exceeds that response caused by replacing cold lithosphere by hot asthenosphere

We start by looking at the thermal contraction history, because we already know how to deal with the isostatic response. Later, we can combine the two into a theoretical statement of subsidence at various steps in the rift and post-rift history.

What we want is to be able to quantify the relationship between time and temperature, if a few physical properties are known.

-- ignore lateral heat flux ==> one-dimensional heat transport by conduction
-- ignore radioactive heat generation within the rocks

WANTED: an equation governing time/space variations of T due to conductive heat transfer:
\[ \rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} \]  (eq. 3 -- see below for insight)

we obtain this equation by equating the energy (units are ML2T-2) needed to maintain a temperature change through time, with the net heat flow through the slab

CONDUCTIVE HEAT TRANSPORT:
1. Fourier equation of heat conduction:

heat flux \( q \) (= flow of heat /unit area and unit time)
\[ q = -k \frac{dT}{dy} \]  (1)

this is one dimensional, in \( y \) direction

\( k = \) coefficient of thermal conductivity (units of energy/unit length and unit time)

for 1-dimensional heat flow, the net heat flow out of a slab of thickness \( dy \) (per unit time and per unit area) is

\[ \frac{\partial^2 T}{\partial y^2} = q(\delta y) - q(y) \]  (1a)

[this is a statement of conservation of energy]
\[ q(\delta y) - q(y) \]  indicates the difference in heat flow on the two sides of the slab]
Problems to interest of us here are time-dependent: at one time heat is introduced, and then it progressively cools. A net heat flow out of our slab \( \delta y \) must reduce its temperature. For this case of time-dependent supply of heat (no radiogenic sources), the basic equation for the conditions to maintain a temperature change at the rate \( \frac{\delta T}{\delta t} \) in a slab \( \delta y \) of unit area:

the energy flow/time needed is

\[
\rho c \frac{\partial T}{\partial t} \delta y
\]  

(2)

where \( c \) = specific heat (energy needed to raise unit mass by one degree)

\( \rho \) = density

Equation 1a expressed general heat flow through a slab, and that heat flow leads to decrease in slab temperature for time-dependent heat introduction. Therefore, if we equate equations 1a & 2 (both are energy/area/time), we get the basic equation governing time/space variatons of \( T \) due to conductive heat transfer:

\[
\frac{\partial^2 T}{\partial y^2} = \frac{k}{\rho c} \frac{\partial T}{\partial t}
\]

This can be rewritten as

\[
\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2}
\]

or

\[
\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2}
\]

(3)

where \( K = \frac{k}{\rho c} \) = thermal diffusivity (units of length\(^2\)/time).

To solve the geological problems of cooling after instantaneous heating, we generalize to 2-dimensions (a semi-infinite half space)

at time \( t = 0 \), \( T_o \) instantaneously established for the body, and its surface is maintained at \( T_{surface} = T_s \)

e.g., \( T_o \) applies to newly introduced oceanic melt, and \( T_s \) is the seawater interface.

if \( T_s < T_o \), the half space cools.

Solve \( \frac{\partial T}{\partial t} \) for boundary conditions:

\( T = T_o \) at \( t = 0 \)

\( T = T_i \) at \( y = 0 \)

\( T \to T_m \) as \( y \to \infty \) (measuring down from the surface)

for oceanic lithosphere, this assumes that all of heat conduction occurs vertically (this will
hold true if lithosphere is relatively thin)
\[ T_0 = T_{mantle} = 1600^\circ K \approx 1300^\circ C. \]

As far as deriving equations goes, this is as far as we are taking it in class. We have shown that you can quantify the relationship between time and temperature, if a few physical properties are known. Those physical properties have been measured in the laboratory for rocks.

So we have a relationship describing the thermal condition of any layer at any point in time.

• This relationship is applied to sedimentary basins in two common ways: predict amount of subsidence per time, and temperature of any horizon through time. The equations describing subsidence are derived from \( \frac{\partial T}{\partial t} \) via

1. the relationship between temperature and density
2. the isostatic feedback to the density changes.

1. The relation between density (\( \rho \)) and temperature:
   \[ d\rho = -\rho \alpha_v dT \]
   where \( \alpha_v \) is the volumetric coefficient of thermal expansion, which is the fractional change in volume with temperature at constant pressure.

Converting to measure of subsidence through time:
final equation is:
\[ \Delta E = \frac{\rho_A}{\rho_A - \rho_{surf}} \left\{ \frac{2\alpha_v(T_{surf} - T_A)}{\pi} \left[ \frac{\sqrt{kt}}{\pi} \right] \right\} \]

This equation is the solution, appropriate to boundary conditions based on cooling of the oceanic lithosphere, for the elevation of the seafloor. It is derived from the expression:
\[ w = 2\rho_m\alpha_v(T_m - T_o)/(\rho_m - \rho_w)\sqrt{\frac{kx}{u_o}} \int_0^\infty \text{erfc} \eta \, d\eta \]

where \( n = (y/2)\sqrt{\frac{u_o}{kx}} \)

\( w = \) depth of ocean floor, \( x = \) horizontal distance from ridge, \( u = \) horizontal velocity of plate motion, \( \alpha_v \) = volume coefficient of thermal expansion; \( k \) is diffusivity, 
\( y = \) vertical dimension, \( y_L \) is the thickness of the lithosphere.

Clearly this full equation involves the thickness of the lithosphere, and thus relates to the mass of the lithosphere. The limit of integration is infinity because, for the specific case of oceanic lithosphere cooling, one can say that \( \rho \to \rho_m \) and \( T \to T_m \) at the base of the lithosphere, which means that the limit on the integral can be changed from \( y = y_L \) to \( y = \infty \).

The definite integral \( \int_0^\infty \text{erfc} \eta \, d\eta \) has the value \( \frac{1}{\pi} \), hence getting us back to the equation:

\[ \Delta E = \frac{\rho_A}{\rho_A - \rho_{\text{surf}}} \left\{ \frac{2\alpha_v(T_{\text{surf}} - T_A)}{\sqrt{\pi t}} \right\} \]

the term in the brackets represents the progressive cooling through time (t), and the constants are: \( \rho_A \) and \( \rho_{\text{surf}} \) are the densities of the asthenosphere and the surface material filling in the subsiding hole, \( T_A \) is the initial temperature of the asthenosphere (~1350°C), \( T_{\text{surf}} \) is the temperature of the material above the lithosphere (water in this case, air in others), \( \alpha \) is the volumetric coefficient of thermal expansion (3.2 \times 10^{-5}/°C), and \( k \) is the thermal diffusivity of the lithosphere (8x10^{-7}m²/sec).

Examples of the validity of the heat transport came first from studies of ocean floor.
The thickness of lithosphere (defined by isotherms) predicted by the heat flow models, then converted to depth of ocean by isostatic compensation, was compared to observation. Depth to ocean floor is a function of square root of time in above equation, and this holds true.

Actually, the heat transport is more complicated for many reasons, some of which are that the sediments have a feedback relationship with the cooling. There are two effects:

1. thermal blanketing (the cold sediments are dropped progressively onto the upper surface, which increases the distance over which the heat conducts or decreases the thermal gradient. Since heat flow rate is related to thermal gradient, the rate of cooling slows down.)

2. the amplification of subsidence by water and sediment loading forces the lower boundary of the cooling half space to subside (beyond what it would do in the open ocean in the absence of sedimentation) -- thus the lower hot area is pushed back into a region where it is in thermal equilibrium (rocks laterally are same temperature), and there is no excess heat loss from that lower part of the system. Thus there is less total heat loss than would have been predicted. (This is a material transfer, whereas the thermal blanketing was something that affected the rate of heat loss).

So equation 3. actually has another term

\[ \frac{\partial}{\partial y} [K(t) \frac{\partial T}{\partial y}] = \rho C \left( \frac{\partial T}{\partial t} + V_y \frac{\partial T}{\partial y} \right) \]  

(3a)

\( V_y \) is the velocity with which lithosphere subsides; note that the thermal conductivity \( k \) changes with time.