"Rheology" = relationship between stress and strain of a material. In the earth, a given stress may result in strain by differing mechanisms, at differing rates, in the various layers. Specifically, a stress applied at the outer surface (a load on the lithosphere) (or laterally at plate boundaries) may generate different responses at several layers of the crust and mantle, or at different places on earth.

Principal types of rheologies of interest to us:

**elastic behavior** --
- stress and strain are linearly related
- strain is recoverable
- strain is time-independent (=instantaneous)

**viscous behavior** --
- strain *rate* is related to stress magnitude
- strain not recoverable
- strain is time-dependent (not instantaneous) (depends on strain rate)
Strength envelope of crust/mantle, showing regions where composition and temperature promote elastic behavior versus other behavior.

"brittle" part of deformation is governed by friction on a set of fractures.
"ductile" part of curve is governed by crystal flow.
TEMPERATURE (°C) vs DEPTH (km) vs MAX SHEAR STRESS (MPa)

- 60 mW/m²
- dry granite
- hz compression
- olivine
- hz extension
Lateral distribution of subsidence due to strength of lithosphere (elastic beam approximation)

the crust and part of the lithosphere distribute the subsidence (or uplift) over a wider area
• result: displacement of an equal volume of asthenosphere, but distributed over a wider zone
• effect: subsidence at the base of the lithosphere is less beneath the load (less than predicted by the Airy calculation) but subsidence continues to the sides ==> produces a broad sedimentary basin

illustration for the case of tectonic shortening, which has thickened the crust:

1. load

   elastic beam (basement not deformed)

2. mountains

   load

   basin
distal uplift (peripheral bulge)

   flexed profile
Demonstration of Flexural Subsidence of thin sheets of differing strengths and rheologies:
cloth or vinyl sheet stretched across circular wooden frame, loaded with sand, subsides in a way that mimics elastic plate bending

Quantifying Flexure

the deflection of the earth's surface due to flexure can be calculated
• equations of traditional mechanics
• must include the effect of the viscous fluid beneath the elastic beam which resists flexure (unlike traditional engineering mechanics)
• to look at first order effects, ignore lateral (in horizontal plane) stresses

(schematic vertical scale)
flexural subsidence is a function of distance:

in two dimensions, point load

new position of reference level

reference level before loading

new position of reference level

The basic equation of flexure of the lithosphere is (for load on infinite, thin plate, who deflection is described by \( w \)):

\[
D \frac{d^4w}{dx^4} + P \frac{d^2w}{dx^2} + (\rho_m - \rho_w)gw = q_a(x) = T(x)\rho \gamma g \tag{3}
\]

the terms have distinct significance:

\( D \frac{d^4w}{dx^4} \), refers to the internal stresses due to bending about mid-line of plate

\( P \frac{d^2w}{dx^2} \), refers to forces applied on the end of the beam (parallel to beam)

\((\rho_m - \rho_i)gw\), is the restoring force applied at the bottom of the plate
\((\rho_m - \rho_i)\) represents the density difference between the material infilling the deformation on the top and bottom of the plate.

\[ q_a(x) = T(x)\rho_T g \] -- the stress applied to top of plate ==> this is the "load" that needs isostatic compensation

where

- \( w \) = vertical deflection
- \( x \) = horizontal direction
- \( D \) = flexural rigidity
- \( P \) = horizontal load = "in-plane" force
- \( \rho_m, \rho_i \) = densities of mantle and material that fills in the surface depression (water, air, or sediment)
- \( g \) = gravity

\[ q_a(x) = (\rho_1 - \rho_2)gt(x) \]

where \((\rho_1 - \rho_2)\) is the density contrast of the surface load, and \( t \) is the thickness of surface load

the flexural rigidity, \( D \),

\[ D = \frac{Eh^3}{12(1-v^2)} \]

where \( E \) is Young's modulus
- \( v \) = Poisson's ratio.
- \( h \) = elastic thickness.

note that flexural rigidity is highly dependent upon the elastic thickness.
major uncertainties/inconsistencies of concern to basin behavior:

- is compensation of isostasy strictly in the mantle, or also in the lower crust?
- how thick is the elastic part of the lithospheric plate?
- does even the "elastic" lithosphere relax stresses (= unflex) over time scale that is important to stratigraphy?

why are these uncertain?

-- observations of flexed lithosphere (topographic data and gravity data) imply that upper lithosphere to ~50 km thickness is frequently strong enough to be elastic
-- but consideration of crustal composition rocks under appropriate stress and temperature conditions imply that crust below ~15 km is very weak (not elastic)
-- some studies in extensional regions with high heat flow and in orogenic belts with extremely thick crust indicate compensation occurs in lower crust (but these are special conditions)
-- because multiple variables control stratigraphy, one possible explanation of some stratigraphic patterns is that basin narrows through time as an initial elastic response decays (=viscoelasticity)
1. Is there a physical analogy of beam in the lithosphere?

Maybe not.

• Models of plate bending vs. observation (topography or gravity) show that the thickness of the bending elastic plate is less than the thickness of the lithosphere. It seems to correspond to rocks that are below a given temperature that controls material behavior: laboratory studies of rheology and oceanic topography suggest a temperature of 450° to 750°C as the upper limit of the RELAXATION ISOTHERM. There is also an upper limit to the elastic lithosphere: it underlies the brittle part of the upper crust (this part’s strain is non-recoverable).

• In various studies of continental lithosphere, the elastic thickness (thickness that corresponds to crust below brittle zone and above relaxation isotherm) ranges from ≈ 5 to 100 km. How much of this range is real is unclear, but some of it should be, to reflect different thermal states of various sites studied.

• "APPARENT elastic thickness" is terminology used to differentiate between the model entity and a material unit.

• These values of elastic thickness are not the entire thickness of the crust and lithosphere.