Convolution

output = input*filter

\[ o(t) = i(t) \ast f(t) \]

or, in discrete terms,
\[ o_t = i_t \ast f_t \]

where \( f_t \) is the filter's impulse response function

s= seismogram
r= reflectivity
w= wavelet

Here \( \ast \) corresponds to mathematical convolution, i.e.

\[ o(t) = \int i(\tau) \cdot f(t-\tau) \, d\tau = \int i(t-\tau) \cdot f(\tau) \, d\tau, \]

or in discrete terms,

\[ o_t = \sum i_{\tau} \cdot f_{t-\tau} \]
Digital Convolution:  A graphic mnemonic

let \( f_t = 4, 3, 2, 1 \)
\( i_t = 2, 0, 1 \)

compute \( o_t = i_t \ast f_t \) by reversing \( i_t \) and cross multiplying with \( f_t \), then summing the products.

i.e.

\[
\begin{array}{cccc}
4 & 3 & 2 & 1 \\
1 & 0 & 2 & \\
1 & 0 & 2 & \\
1 & 0 & 2 & \\
1 & 0 & 2 & \\
1 & 0 & 2 & \\
\end{array}
\]

\[
\begin{array}{cccc}
& 2 \times 4 = & 8 \\
& 4 \times 0 + 2 \times 3 = & 6 \\
& 4 \times 1 + 3 \times 0 + 2 \times 2 = & 8 \\
& 3 \times 1 + 2 \times 0 + 2 \times 1 = & 5 \\
& 2 \times 1 + 1 \times 0 = & 2 \\
& 1 \times 1 & 1 \\
\end{array}
\]

\( o_t = 8, 6, 8, 5, 2, 1 \)

Note: The seismic trace can be cast as a convolution where

\[
s_t = \text{seismic trace (geophone signal)}
\]
\[
r_t = \text{reflection coefficients (as function of travel time)}
\]
\[
w_t = \text{seismic wavelet (source signal)}
\]

\[
s_t = r_t \ast w_t
\]

Here the earth \( (r_t) \) acts as a filter on the seismic input signal \( (w_t) \).